Razlika kvadrata

Binom oblika $A^2 - B^2$ nazivamo **razlika kvadrata**. Naprimjer: $x^2 - y^2$, zatim $(4x)^2 - z^2$ itd. Razlika kvadrata se dobija množenjem razlike i zbira dva ista broja, tj.
$(A - B) \cdot (A + B) = A^2 - B^2$

Provjerimo tu tvrdnju:
$(A - B) \cdot (A + B) = A^2 + AB - AB - B^2 = A^2 - B^2$

Koristeći formulu za **razliku kvadrata** $(A - B) \cdot (A + B) = A^2 - B^2$ izračunaj (odredi proizvod):

1) $(x - y)(x + y) = x^2 - y^2$

2) $(a - b)(a + b) = a^2 - b^2$

3) $(2a - 1)(2a + 1) = (2a)^2 - 1^2 = 2^2 a^2 - 1^2 = 4a^2 - 1$

4) $(x - 1)(x + 1) = x^2 - 1^2 = x^2 - 1$

5) $(x - 2)(x + 2) = x^2 - 2^2 = x^2 - 4$

6) $(x - 3)(x + 3) = x^2 - 3^2 = x^2 - 9$

7) $(3a + b)(3a - b) = (3a)^2 - b^2 = 3^2 a^2 - b^2 = 9a^2 - b^2$

8) $(2a - 3b)(2a + 3b) = (2a)^2 - (3b)^2 = 2^2 a^2 - 3^2 b^2 = 4a^2 - 9b^2$

9) $(2xy - 5z)(2xy + 5z) = (2xy)^2 - (5z)^2 = 2^2 x^2 y^2 - 5^2 z^2 = 4x^2 y^2 - 25z^2$

10) $(4x + 3yz)(4x - 3yz) = (4x)^2 - (3yz)^2 = 4^2 x^2 - 3^2 y^2 z^2 = 16x^2 - 9y^2 z^2$

11) $(3ba - 7c)(3ba + 7c) = (3ba)^2 - (7c)^2 = 3^2 b^2 a^2 - 7^2 c^2 = 9b^2 a^2 - 49c^2$

12) $(5xy - x)(5xy + x) = (5xy)^2 - x^2 = 5^2 x^2 y^2 - x^2 = 25x^2 y^2 - x^2$

13) $\left(\dfrac{1}{2}x + y\right)\left(\dfrac{1}{2}x - y\right) = \left(\dfrac{1}{2}x\right)^2 - y^2 = \left(\dfrac{1}{2}\right)^2 x^2 - y^2 = \dfrac{1^2}{2^2}x^2 - y^2 = \dfrac{1}{4}x^2 - y^2$

14) $\left(\dfrac{2}{3} - x\right)\left(\dfrac{2}{3} + x\right) = \left(\dfrac{2}{3}\right)^2 - x^2 = \dfrac{2^2}{3^2} - x^2 = \dfrac{4}{9} - x^2$

15) $\left(5a - \dfrac{3}{4}b\right)\left(5a + \dfrac{3}{4}b\right) = (5a)^2 - \left(\dfrac{3}{4}b\right)^2 = 5^2 a^2 - \left(\dfrac{3}{4}\right)^2 b^2 = 25a^2 - \dfrac{9}{16}b^2$

16) $\left(\dfrac{3}{4}xy - 2z\right)\left(\dfrac{3}{4}xy + 2z\right) = \left(\dfrac{3}{4}xy\right)^2 - (2z)^2 = \left(\dfrac{3}{4}\right)^2 x^2 y^2 - 2^2 z^2 = \dfrac{9}{16}x^2 y^2 - 4z^2$

17) $\left(\dfrac{2}{3}x^2 - 5y^4 z^5\right)\left(\dfrac{2}{3}x^2 - 5y^4 z^5\right) = \left(\dfrac{2}{3}x^2\right)^2 - \left(5y^4 z^5\right)^2 = \left(\dfrac{2}{3}\right)^2 (x^2)^2 - 5^2 (y^4)^2 (z^5)^2 =$

$= \dfrac{4}{9}x^4 - 25y^8 z^{10}$

18) $\left(\dfrac{1}{2}x^5 + \dfrac{3}{yz}\right)\left(\dfrac{1}{2}x^5 - \dfrac{3}{yz}\right) = \left(\dfrac{1}{2}x^5\right)^2 - \left(\dfrac{3}{yz}\right)^2 = \left(\dfrac{1}{2}\right)^2 (x^5)^2 - \dfrac{3^2}{(yz)^2} = \dfrac{1}{4}x^{10} - \dfrac{9}{y^2 z^2}$

19) $\left(\dfrac{4}{5}x^2 - \dfrac{5}{7y^3 z^4}\right)\left(\dfrac{4}{5}x^2 + \dfrac{5}{7y^3 z^4}\right) = \left(\dfrac{4}{5}x^2\right)^2 - \left(\dfrac{5}{7y^3 z^4}\right)^2 = \left(\dfrac{4}{5}\right)^2 (x^2)^2 - \dfrac{5^2}{\left(7y^3 z^4\right)^2} =$

$= \dfrac{4^2}{5^2}x^{2\cdot 2} - \dfrac{5^2}{7^2 \cdot (y^3)^2 \cdot (z^4)^2} = \dfrac{16}{25}x^4 - \dfrac{25}{49\cdot y^{3\cdot 2} \cdot z^{4\cdot 2}} = \dfrac{16}{25}x^4 - \dfrac{25}{49y^6 z^8}$

20) $\left(2\dfrac{2}{3}x - y\right)\left(2\dfrac{2}{3}x + y\right) = \left(2\dfrac{2}{3}x\right)^2 - y^2 = \left(\dfrac{8}{3}x\right)^2 - y^2 = \left(\dfrac{8}{3}\right)^2 \cdot x^2 - y^2 = \dfrac{8^2}{3^2}x^2 - y^2 =$

$= \dfrac{64}{9}x^2 - y^2$

21) $\left(1\dfrac{1}{2}x^2 - \dfrac{1}{3}y\right)\left(1\dfrac{1}{2}x^2 + \dfrac{1}{3}y\right) = \left(1\dfrac{1}{2}x^2\right)^2 - \left(\dfrac{1}{3}y\right)^2 = \left(\dfrac{3}{2}x^2\right)^2 - \left(\dfrac{1}{3}y\right)^2 = \left(\dfrac{3}{2}\right)^2 \cdot (x^2)^2 - \left(\dfrac{1}{3}\right)^2 \cdot y^2 =$

$= \dfrac{3^2}{2^2} \cdot x^{2\cdot 2} - \dfrac{1^2}{2^2} \cdot y^2 = \dfrac{9}{4}x^4 - \dfrac{1}{4}y^2$

$22) \left(x^4 - 0,75\right)\left(x^4 + 0,75\right) = \left(x^4\right)^2 - 0,75^2 = x^{4\cdot 2} - \left(\dfrac{75}{100}\right)^2 = x^8 - \left(\dfrac{3}{4}\right)^2 = x^8 - \dfrac{9}{16}$

$23) \left(a - b^2\right)\left(a + b^2\right) = a^2 - \left(b^2\right)^2 = a^2 - b^{2\cdot 2} = a^2 - b^4$

$24) \left(3 - a^2\right)\left(3 + a^2\right) = 3^2 - \left(a^2\right)^2 = 9 - a^4$

$25) \left(x^2 - y^3\right)\left(x^2 + y^3\right) = \left(x^2\right)^2 - \left(y^3\right)^2 = x^{2\cdot 2} - y^{3\cdot 2} = x^4 - y^6$

$26) \left(7x^2 - 5y^3\right)\left(7x^2 + 5y^3\right) = \left(7x^2\right)^2 - \left(5y^3\right)^2 = 7^2 \cdot \left(x^2\right)^2 - 5^2 \cdot \left(y^3\right)^2 = 49x^4 - 25y^6$

$27) \left(5x^7 - y^2\right)\left(5x^7 + y^2\right) = \left(5x^7\right)^2 - \left(y^2\right)^2 = 5^2 \cdot \left(x^7\right)^2 - \left(y^2\right)^2 = 25x^{7\cdot 2} - y^{2\cdot 2} = 25x^{14} - y^4$

$28) \left(2x^2 y^3 - 3z^4\right)\left(2x^2 y^3 + 3z^4\right) = \left(2x^2 y^3\right)^2 - \left(3z^4\right)^2 = 2^2 \cdot \left(x^2\right)^2 \cdot \left(y^3\right)^2 - 3^2 \cdot \left(z^4\right)^2 =$
$= 4 \cdot x^{2\cdot 2} \cdot y^{3\cdot 2} - 9 \cdot z^{4\cdot 2} = 4x^4 y^6 - 9z^8$

$29) \left(3x^3 y - 3z^2\right)\left(3x^3 y + 3z^2\right) = \left(3x^3 y\right)^2 - \left(3z^2\right)^2 = 3^2 \cdot \left(x^3\right)^2 \cdot y^2 - 3^2 \cdot \left(z^2\right)^2 =$
$= 9 \cdot x^{3\cdot 2} \cdot y^2 - 9z^{2\cdot 2} = 9x^6 y^2 - 9z^4$

$30) \left(9x^5 - 11y^3 z\right)\left(9x^5 + 11y^3 z\right) = \left(9x^5\right)^2 - \left(11y^3 z\right)^2 = 9^2 \cdot \left(x^5\right)^2 - 11^2 \cdot \left(y^3\right)^2 \cdot z^2 =$
$= 81 \cdot x^{5\cdot 2} - 121 \cdot y^{3\cdot 2} \cdot z^2 = 81x^{10} - 121y^6 z^2$

$31) \left(3x^2 y^3 z^4 - 4a^5 b^6\right)\left(3x^2 y^3 z^4 + 4a^5 b^6\right) = \left(3x^2 y^3 z^4\right)^2 - \left(4a^5 b^6\right)^2 =$
$= 3^2 \cdot \left(x^2\right)^2 \cdot \left(y^3\right)^2 \cdot \left(z^4\right)^2 - 4^2 \cdot \left(a^5\right)^2 \cdot \left(b^6\right)^2 = 9 \cdot x^{2\cdot 2} \cdot y^{3\cdot 2} \cdot z^{4\cdot 2} - 16 \cdot a^{5\cdot 2} \cdot b^{6\cdot 2} =$
$= 9x^4 y^6 z^8 - 16a^{10} b^{12}$

$32) \left(a^x + a^y\right)\left(a^x - a^y\right) = \left(a^x\right)^2 - \left(a^y\right)^2 = a^{x\cdot 2} - a^{y\cdot 2} = a^{2x} - a^{2y}$

$33) \left(2a^x - 3b^y\right)\left(2a^x + 3b^y\right) = \left(2a^x\right)^2 - \left(3b^y\right)^2 = 2^2 \cdot \left(a^x\right)^2 - 3^2 \cdot \left(b^y\right)^2 = 4a^{2x} - 9b^{2y}$

$33) \left(2^m - 2^n\right)\left(2^m - 2^n\right) = \left(2^m\right)^2 - \left(2^n\right)^2 = 2^{m\cdot 2} - 2^{n\cdot 2} = 2^{2m} - 2^{2n}$

$34) \left(3^m - 2^n\right)\left(3^m - 2^n\right) = \left(3^m\right)^2 - \left(2^n\right)^2 = 3^{m\cdot 2} - 2^{n\cdot 2} = 3^{2m} - 2^{2n}$

34) $\left(3^n - 2^m\right)\left(3^n - 2^m\right) = \left(3^n\right)^2 - \left(2^m\right)^2 = 3^{2n} - 2^{2m}$

35) $(x + y - 1)(x + y + 1) = [(x + y) - 1] \cdot [(x + y) + 1] = (x + y)^2 - 1^2 = x^2 + 2xy + y^2 - 1$

36) $(x + y - 1)(x - y + 1) = [x + (y - 1)] \cdot [x - (y - 1)] = x^2 - (y - 1)^2 = x^2 - \left(y^2 - 2y + 1\right) =$
$= x^2 - y^2 + 2y - 1$

37) $(3x + y - z)(3x - y + z) = [3x + (y - z)] \cdot [3x - (y - z)] = (3x)^2 - (y - z)^2 =$
$= 9x^2 - \left(y^2 - 2yz + z^2\right) = 9x^2 - y^2 + 2yz - z^2 = 9x^2 - y^2 - z^2 + 2yz$

38) $(2x - 3y + 5z)(2x + 3y - 5z) = [2x - (3y - 5z)] \cdot [2x + (3y - 5z)] = (2x)^2 - (3y - 5z)^2 =$
$= 2^2 \cdot x^2 - \left((3y)^2 - 2 \cdot 3y \cdot 5z + (5z)^2\right) = 4x^2 - \left(3^2 \cdot y^2 - 30yz + 5^2 \cdot z^2\right) =$
$= 4x^2 - \left(9y^2 - 30yz + 25z^2\right) = 4x^2 - 9y^2 + 30yz - 25z^2$

39) $\left(2x^2 + 3y^3 + 5z^6\right)\left(2x^2 + 3y^3 - 5z^6\right) = \left[\underbrace{\left(2x^2 + 3y^3\right)}_{A} + \underbrace{5z^6}_{B}\right] \cdot \left[\underbrace{\left(2x^2 + 3y^3\right)}_{A} - \underbrace{5z^6}_{B}\right] =$

$\underbrace{\left(2x^2 + 3y^3\right)^2}_{A\,na\,2} - \underbrace{\left(5z^6\right)^2}_{B\,na\,2} = \left(2x^2\right)^2 + 2 \cdot 2x^2 \cdot 3y^3 + \left(3y^3\right)^2 - 5^2 \cdot \left(z^6\right)^2 =$

$= 2^2 \cdot \left(x^2\right)^2 + 12x^2 y^3 + 3^2 \cdot \left(y^3\right)^2 - 25z^{6 \cdot 2} = 4x^{2 \cdot 2} + 12x^2 y^3 + 9y^{3 \cdot 2} - 25z^{12} =$
$= 4x^4 + 12x^2 y^3 + 9y^6 - 25z^{12}$

40) $\left(5x - 4y^2 + z^3\right)\left(5x + 4y^2 - z^3\right) = \left[5x - \left(4y^2 - z^3\right)\right]^2 - \left[5x + \left(4y^2 - z^3\right)\right]^2 =$
$= (5x)^2 - \left(4y^2 - z^3\right)^2 = 5^2 \cdot x^2 - \left(\left(4y^2\right)^2 - 2 \cdot 4y^2 \cdot z^3 + \left(z^3\right)^2\right) =$
$25x^2 - \left(4^2 \cdot \left(y^2\right)^2 - 8y^2 z^3 + \left(z^3\right)^2\right) = 25x^2 - \left(16y^4 - 8y^2 z^3 + z^6\right) = 25x^2 - 16y^4 + 8y^2 z^3 - z^6$

41) $48 \cdot 52 = (50 - 2)(50 + 2) = 50^2 - 2^2 = 2500 - 4 = 2496$

42) $97 \cdot 103 = (100 - 3)(100 + 3) = 100^2 - 3^2 = 10000 - 9 = 9991$

43) $195 \cdot 205 = (200 - 5)(200 + 5) = 200^2 - 5^2 = 40000 - 25 = 39975$

44) $\left(\sqrt{7} + \sqrt{5}\right)\left(\sqrt{7} - \sqrt{5}\right) = \left(\sqrt{7}\right)^2 - \left(\sqrt{5}\right)^2 = 7 - 5 = 2$

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45)

$45)\sqrt{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}=\sqrt{\left(2+\sqrt{3}\right)\cdot\left(2-\sqrt{3}\right)}=\sqrt{2^2-\left(\sqrt{3}\right)^2}=\sqrt{4-3}=\sqrt{1}=1$

$46)\sqrt{\sqrt{65}-8}\cdot\sqrt{\sqrt{65}+8}=\sqrt{\left(\sqrt{65}\right)^2-8^2}=\sqrt{65-64}=\sqrt{1}=1$

$47)(2a-1)^2+(4a+1)^2+(2a+1)(2a-1)=(2a)^2-2\cdot2a\cdot1+1^2+(4a)^2+2\cdot4a\cdot1+1^2+$
$+(2a)^2-1^2=4a^2-4a+1+16a^2+8a+1+4a^2-1=24a^2+4a+1$

$48)(3x+11)(3x-11)-(x-3)^2+(2x+1)^2=(3x)^2-11^2-\left(x^2-6x+9\right)+4x^2+4x+1=$
$=9x^2-121-x^2+6x-9+4x^2+4x+1=12x^2+10x-129$

$49)(2a+1)(2a-1)(4a^2+1)=\underbrace{\left((2a)^2-1^2\right)}_{razlika..kvadrata}(4a^2+1)=\underbrace{\left(4a^2-1\right)\left(4a^2+1\right)}_{razlika..kvadrata}=\left(4a^2\right)^2-1^2=$
$=4^2\cdot\left(a^2\right)^2-1=16a^4-1$

$50)(1-2abc)(1+2abc)(1+4a^2b^2c^2)=\underbrace{\left(1^2-(2abc)^2\right)}_{razlika..kvadrata}(1+4a^2b^2c^2)=$
$=\underbrace{\left(1-4a^2b^2c^2\right)\left(1+4a^2b^2c^2\right)}_{razlika..kvadrata}=1^2-\left(4a^2b^2c^2\right)^2=1-4^2\cdot\left(a^2\right)^2\cdot\left(b^2\right)^2\cdot\left(c^2\right)^2=1-16a^4b^4c^4$

$51)3\cdot\underbrace{(2a-3)^2}_{kvadrat..binoma}-\underbrace{(3a-1)(3a+1)}_{razlika..kvadrata}+\underbrace{(3a+1)^2}_{kvadrat..binoma}=$
$=3\cdot\left(4a^2-12a+9\right)-\left((3a)^2-1^2\right)+9a^2+6a+1=$
$=12a^2-36a+27-9a^2+1+9a^2+6a+1=12a^2-30a+29$

$52)(1-a^2b^3)(x^2y+k)(k-x^2y)(a^2b^3+1)=(1-a^2b^3)(1+a^2b^3)(x^2y+k)(x^2y-k)(-1)$
$=\left(1^2-\left(a^2b^3\right)^2\right)\left(\left(x^2y\right)^2-k^2\right)=\left(1-a^4b^6\right)\left(x^4y^2-k^2\right)$

Zadaci za samostalan rad učenika

Koristeći formulu za **razliku kvadrata** $(A - B) \cdot (A + B) = A^2 - B^2$ izračunaj (odredi proizvod):

1) $(x - a)(x + a) =$

2) $(2x - 1)(2x + 1) =$

3) $(3x - 5)(3x + 5) =$

4) $(7y - 6)(7y + 6) =$

5) $(0{,}1x + 0{,}2)(0{,}1x - 0{,}2) =$

6) $\left(1\frac{1}{3}x - 2\frac{2}{5}y\right)\left(1\frac{1}{3}x + 2\frac{2}{5}y\right) =$

7) $(0{,}1x^3 - 0{,}3y)(0{,}1x^3 + 0{,}3y) =$

8) $(x^3 + y^5)(x^3 - y^5) =$

9) $(4x^2 - 3y^3)(4x^2 + 3y^3) =$

10) $(2ab^2 + c^3)(2ab^2 - c^3) =$

12) $(x^n + y^m)(x^n - y^m) =$

13) $(x^{n+1} - y^{m-2})(x^{n+1} + y^{m-2}) =$

14) $(a + b + c)(a - b - b - c) =$

15) $(x + 2y - z)(x - 2y - z) =$

16) $(x - 2y - 8z)(x + 2y - 8z) =$