Kvardrat binoma

Koristeći se formulama za **kvadrat zbira** i **kvadrat razlike**:
$(A+B)^2 = A^2 + 2\cdot A\cdot B + B^2$ (1) i $(A-B)^2 = A^2 - 2\cdot A\cdot B + B^2$ (2) izračunaj:

1) $(x+1)^2 = x^2 + 2\cdot x\cdot 1 + 1^2 = x^2 + 2x + 1$
Objašnjenje: A=x , B=1,formula (1)

2) $(x-1)^2 = x^2 - 2\cdot x\cdot 1 + 1^2 = x^2 + 2x + 1$
Objašnjenje: A=x , B=1 , formula (2)

3) $(x+y)^2 = x^2 + 2xy + y^2$

4) $(x-y)^2 = x^2 - 2xy + y^2$

5) $(x+3)^2 = x^2 + 2\cdot x\cdot 3 + 3^2 = x^2 + 6x + 9$

Objašnjenje: $A = x, B = 3$, *formula*(1)

6) $(x-5)^2 = x^2 - 2\cdot x\cdot 5 + 5^2 = x^2 - 10 + 25$

7) $(2-x)^2 = 2^2 - 2\cdot 2\cdot x + x^2 = 4 - 4x + x^2 = x^2 - 4x + 4$

8) $(7-x)^2 = 7^2 - 2\cdot 7\cdot x + x^2 = 49 - 14x + x^2 = x^2 - 14x + 49$

9) $(x+5y)^2 = x^2 + 2\cdot x\cdot 5y + (5y)^2 = x^2 + 10xy + 5^2 y^2 = x^2 + 10xy + 25y^2$

Napomena:Primijetimo da je u posljednjem zadatku primijenjeno pravilo: $(a\cdot b)^n = a^n \cdot b^n$.
U našem slučaju imali smo: $(5\cdot y)^2 = 5^2 \cdot y^2 = 25y^2$

10) $(5x-y)^2 = (5x)^2 - 2\cdot 5x\cdot y + y^2 = 5^2 x^2 - 10xy + y^2 = 25x^2 - 10xy + y^2$

Isto to pravilo smo ovdje primijenili.

11) $(7x+y)^2 = (7x)^2 + 2\cdot 7x\cdot y + y^2 = 7^2 x^2 + 2\cdot 7x\cdot y + y^2 = 49x^2 + 14xy + y^2$

12) $(x-3y)^2 = x^2 - 2\cdot x\cdot 3y + (3y)^2 = x^2 - 6xy + 3^2 y^2 = x^2 - 6xy + 9y^2$

Jasno je da je uz malo vještine,naprimjer u zadnjem zadatku, dovoljno napisati i ovako (skraćeno):
$(x-3y)^2 = x^2 - 2\cdot x\cdot 3y + (3y)^2 = x^2 - 6xy + 9y^2$

13) $(3x+2) = (3x)^2 + 2\cdot 3x\cdot 2 + 2^2 = 9x^2 + 12x + 4$

14) $(2a-5b)^2 = (2a)^2 - 2\cdot 2a\cdot 5b + (5b)^2 = 2^2 a^2 - 20ab + 5^2 b^2 = 4a^2 - 20ab + 25b^2$

Objašnjenje:A=2a , B=5b , formula (2)

U 14 zadatku smo, također,primijenili pravilo: $(a\cdot b)^n = a^n\cdot b^n$.

15) $(2x+3y)^2 = (2x)^2 + 2\cdot 2x\cdot 3y + (3y)^2 = 2^2x^2 + 12xy + 3^2y^2 = 4x^2 + 12xy + 9y^2$

16) $(5x+2y)^2 = (5x)^2 + 2\cdot 5x\cdot 2y + (2y)^2 = 5^2x^2 + 20xy + 2^2y^2 = 25x^2 + 20xy + 4y^2$

17) $(3x+4y)^2 = (3x)^2 + 2\cdot 3x\cdot 4y + (4y)^2 = 3^2x^2 + 24xy + 4^2y^2 = 9x^2 + 24xy + 16y^2$

18) $(3m+n)^2 = (3m)^2 + 2\cdot 3m\cdot n + n^2 = 9m^2 + 6mn + n^2$

19) $(2m-5n)^2 = (2m)^2 - 2\cdot 2m\cdot 5n + (5n)^2 = 4m^2 - 20mn + 25n^2$

20) $(0,2x+3)^2 = (0,2x)^2 + 2\cdot 0,2x\cdot 3 + 3^2 = 0,2^2x^2 + 1,2x + 9 = 0,04x^2 + 1,2x + 9$

21) $(1,2x+0,3)^2 = 1,44x^2 + 0,72x + 0,09$

22) $(0,5x-2y)^2 = (0,5x)^2 - 2\cdot 0,5x\cdot 2y + (2y)^2 = 0,25x^2 - 2xy + 4y^2$

23) $\left(x+\frac{1}{2}\right)^2 = x^2 + 2\cdot x\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 = x^2 + x + \frac{1}{4}$

24) $\left(\frac{3}{2}x+1\right)^2 = \left(\frac{3}{2}x\right)^2 + 2\cdot\frac{3}{2}x\cdot 1 + 1^2 = \left(\frac{3}{2}\right)^2x^2 + 3x + 1 = \frac{9}{4}x^2 + 3x + 1$

25) $\left(\frac{3}{4}x+\frac{2}{3}y\right)^2 = \left(\frac{3}{4}x\right)^2 + 2\cdot\frac{3}{4}x\cdot\frac{2}{3}y + \left(\frac{2}{3}y\right)^2 = \left(\frac{3}{4}\right)^2\cdot x^2 + xy + \left(\frac{2}{3}\right)^2\cdot y^2 = \frac{9}{16}x^2 + xy + \frac{4}{9}y^2$

26) $\left(\frac{5}{2}x+0,2y\right)^2 = \left(\frac{5}{2}x\right)^2 + 2\cdot\frac{5}{2}x\cdot 0,2y + (0,2y)^2 = \left(\frac{5}{2}\right)^2\cdot x^2 + 2\cdot\frac{5}{2}x\cdot\frac{2}{10}y + (0,2)^2y^2$

$= \frac{25}{4}x^2 + \frac{20}{20}xy + \left(\frac{2}{10}\right)^2y^2 = \frac{25}{4}x^2 + xy + \left(\frac{1}{5}\right)^2y^2 = \frac{25}{4}x^2 + xy + \frac{1}{25}y^2$

27) $\left(4x-\frac{3}{2}y\right)^2 = (4x)^2 - 2\cdot 4x\cdot\frac{3}{2}y + \left(\frac{3}{2}y\right)^2 = 4^2x^2 - 8x\cdot\frac{3}{2}y + \left(\frac{3}{2}\right)^2y^2 = 16x^2 - \frac{24xy}{2} + \frac{9}{4}y^2 =$

$= 16x^2 - 12xy + \frac{9}{4}y^2$

28) $\left(\frac{1}{2}x+\frac{2}{3}y^3\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\cdot\frac{1}{2}x\cdot\frac{2}{3}y^3 + \left(\frac{2}{3}y^3\right)^2 = \left(\frac{1}{2}\right)^2\cdot x^2 + \frac{2}{3}xy^3 + \left(\frac{2}{3}\right)^2\cdot\left(y^3\right)^2 =$

$= \frac{1}{4}x^2 + \frac{2}{3}xy^3 + \frac{4}{9}y^6$

Napomena:Primijetimo da smo u posljednjem zadatku koristili pravilo stepenovanja stepena: $\left(A^m\right)^n = A^{m\cdot n}$

U našem slučaju imali smo: $A = y^3$; $\left(y^3\right)^2 = y^{3\cdot 2} = y^6$

$$29)\left(\frac{2}{3}x^3-\frac{3}{4}y^4\right)^2=\left(\frac{2}{3}x^3\right)^2-2\cdot\frac{2}{3}x^3\cdot\frac{3}{4}y^4+\left(\frac{3}{4}y^4\right)^2=\left(\frac{2}{3}\right)^2\cdot\left(x^3\right)^2-2\cdot\frac{2}{3}x^3\cdot\frac{3}{4}y^4+\left(\frac{3}{4}\right)^2\cdot\left(y^4\right)^2$$
$$=\frac{4}{9}x^6-x^3y^4+\frac{9}{16}y^8$$
$$30)\left(\frac{1}{4}xz-\frac{3}{2}y\right)^2=\left(\frac{1}{4}xz\right)^2-2\cdot\frac{1}{4}xz\cdot\frac{3}{2}y+\left(\frac{3}{2}y\right)^2=\frac{1}{16}x^2z^2-\frac{3}{4}xzy+\frac{9}{4}y^2$$
$$31)\left(3-x^4\right)^2=3^2-2\cdot3\cdot x^4+\left(x^4\right)^2=9-6x^4+x^8$$
$$32)\left(3x^4+7y^5\right)^2=\left(3x^4\right)^2+2\cdot\left(3x^4\right)\cdot\left(7y^5\right)+\left(7y^5\right)^2=3^2\left(x^4\right)^2+2\cdot3x^4\cdot7y^5+\left(7y^5\right)^2$$
$$=9x^{4\cdot2}+42x^4y^5+49y^{5\cdot2}=9x^8+42x^4y^5+49y^{10}$$
$$33)\left(2x^2y^3-3z^2\right)^2=\left(2x^2y^3\right)^2-2\cdot2x^3y^3\cdot3z^2+\left(3z^2\right)^2=4x^4y^6-12x^2y^3z^2+9z^4$$
$$34)(x+4y)^{-2}=\frac{1}{(x+4y)^2}=\frac{1}{x^2+2\cdot x\cdot4y+(4y)^2}=\frac{1}{x^2+8xy+16y^2}$$

Napomena:U posljednjem zadatku koristili smo pravilo: $A^{-m}=\frac{1}{A^m}$
U našem slučaju bilo je :A=x+4y

$$35)\left(2a^x+3b^y\right)^2=\left(2a^x\right)^2+2\cdot2a^x\cdot3b^y+\left(3b^y\right)^2=2^2\cdot\left(a^x\right)^2+12a^xb^y+3^2\cdot\left(b^y\right)^2=$$
$$=4a^{x\cdot2}+12a^xb^y+9b^{y\cdot2}=4a^{2x}+12a^xb^y+9b^{2y}$$
$$36)\left(2^{m+1}+2^{m-1}\right)^2=\left(2^{m+1}\right)^2+2\cdot2^{m+1}\cdot2^{m-1}+\left(2^{m-1}\right)^2=2^{(m+1)\cdot2}+2^1\cdot2^{m+1}\cdot2^{m-1}+2^{(m-1)\cdot2}=$$
$$=2^{2m+2}+2^{1+m+1+m-1}+2^{2m-2}=2^{2m+2}+2^{2m+1}+2^{2m-2}=4^{m+1}+2^{2m+1}+4^{m-1}$$
$$37)(-x-3)^2=\left(-1\cdot(x+3)\right)^2=(-1)^2\cdot(x+3)^2=x^2+6x+9$$
$$38)\left(-2x^2y^3+3z^4\right)^2=\left(3z^4-2x^2y^3\right)^2=\left(3z^4\right)^2-2\cdot3z^4\cdot2x^2y^3+\left(2x^2y^3\right)^2=$$
$$=3^2\cdot\left(z^4\right)^2-12x^2y^3z^4+2^2\cdot\left(x^2\right)^2\cdot\left(y^3\right)^2=9z^{4\cdot2}-12x^2y^3z^4+4x^{2\cdot2}y^{3\cdot2}=$$
$$=9z^8-12x^2y^3z^4+4x^4y^6=4x^4y^6-12x^2y^3z^4+9z^8$$

3Izračunati:
$$39)(x+y+z)^2=\left[(x+y)+z\right]^2=(x+y)^2+2\cdot(x+y)\cdot z+z^2=x^2+2xy+y^2+2xy+2yz+z^2=$$
$$=x^2+y^2+z^2+2xy+2xz+2yz$$
$$40)\left(x^2+2y+2xz\right)^2=$$
Koristeći rezultat predhodnog zadatka dobijamo:

$$\left(x^2+2y+2xz\right)^2=\left(x^2\right)^2+(2y)^2+(2xz)^2+2\cdot x^2\cdot2y+2\cdot x^2\cdot2xz+2\cdot2y\cdot2xz=$$
$$=x^4+4y^2+4x^2z^2+4x^2y+4x^3z+8xyz$$

Zadaci za samostalan rad učenika:

1) $(3x-7y)^2 =$

2) $\left(0{,}4x-\frac{5}{3}y\right)=$

3) $\left(1\frac{1}{2}x+y\right)^2 =$

4) $\left(2\frac{2}{3}x-1\frac{1}{4}y\right)=$

5) $(x^2+1)^2 =$

6) $(x^2-2)^2 =$

7) $(3a^2-b^3)^2 =$

8) $(2a^3-b^2)^2 =$

9) $(a^3-5b^2)^2 =$

10) $(2x^3+3y^3)^2 =$

11) $(5x-6y)^{-2} =$

12) $\left(-\frac{1}{2}x-4y^2\right)^2 =$

13) $(-5a^2b^3+2c^4)^2 =$

14) $(2^m+3^n)^2 =$

15) $(2^n+2^{1-n})^2 =$

16) $(2a^x-3b^x)^2 =$

Rješenje zadataka za samostalan rad:

1) $9x^2 - 42xy + 49y^2$

2) $\frac{4}{25}x^2 - \frac{4}{3}xy + \frac{25}{9}y^2$

3) $\frac{9}{4}x^2 + xy + y^2 =$

4) $\frac{64}{9}x^2 - \frac{40}{6}xy + \frac{25}{16}y^2$

5) $x^4 + 2x^2 + 1$

6) $x^4 - 4x^2 + 4$

7) $9a^4 - 6a^2b^3 + b^6$

8) $4a^6 - 4a^3b^2 + b^4$

9) $a^6 - 10a^3b^2 + 25b^4$

10) $4x^6 + 12x^3y^3 + 9y^6$

11) $\frac{1}{25x^2 - 60xy + 36y^2}$

12) $\frac{1}{4}x^2 + 4xy^2 + 16y^4$

13) $25a^4b^6 - 20a^2b^3c^4 + 4c^8$

14) $4^n + 2^{m+1} \cdot 3^n + 9^n$

15) $4^n - 4 + 4^{1-n}$

16) $4a^{2x} - 12a^x b^x + 9b^{2x}$