MATEMATIKA .BA

Kub binoma

Koristeći se formulama za **kub zbira**: $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$ i **kub razlike:** $(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$ izračunati:

$1)(x+y)^3 = x^3 + 3\cdot x^2 \cdot y + 3\cdot x \cdot y^2 + y^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$2)(x-y)^3 = x^3 - 3\cdot x^2 \cdot y + 3\cdot x \cdot y^2 - y^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$3)(y-x)^3 = y^3 - 3\cdot y^2 \cdot x + 3\cdot y \cdot x^2 - x^3 = y^3 - 3xy^2 + 3x^2y - x^3$

$4)(x-1)^3 = x^3 - 3\cdot x^2 \cdot 1 + 3\cdot x \cdot 1^2 - 1^3 = x^3 - 3x^2 + 3x - 1$

$5)(1-x)^3 = 1^3 - 3\cdot 1^2 \cdot x + 3\cdot 1 \cdot x^2 - x^3 = 1 - 3x + 3x^2 - x^3$

$6)(x+1)^3 = x^3 + 3\cdot x^2 \cdot 1 + 3\cdot x \cdot 1^2 + 1^3 = x^3 + 3x^2 + 3x + 1$

$7)(x-2)^3 = x^3 - 3\cdot x^2 \cdot 2 + 3\cdot x \cdot 2^2 - 2^3 = x^3 - 6x^2 + 12x - 8$

$8)(2-x)^3 = 2^3 - 3\cdot 2^2 \cdot x + 3\cdot 2 \cdot x^2 - x^3 = 8 - 12x + 6x^2 - x^3$

$9)(x+3)^3 = x^3 + 3\cdot x^2 \cdot 3 + 3\cdot x \cdot 3^2 + 3^3 = x^3 + 9x^2 + 27x + 27$

$10)(a+2b)^3 = a^3 + 3\cdot a^2 \cdot 2b + 3\cdot a \cdot (2b)^2 + (2b)^3 = a^3 + 6a^2b + 3a\cdot 4b^2 + 8b^3 =$
$= a^3 + 6a^2b + 12ab^2 + 8b^3$

Napomena:U posljednjem zadatku koristili smo pravilo: $(a\cdot b)^n = a^n \cdot b^n$
U našem slučaju imali smo: $(2b)^2 = 2^2 \cdot b^2 = 4b^2$ i $(2b)^3 = 2^3 \cdot b^3 = 8b^3$ (zad.10)

$11)(2a-3b)^3 = (2a)^3 - 3\cdot(2a)^2 \cdot 3b + 3\cdot 2a \cdot (3b)^2 - (3b)^3 = 8a^3 - 3\cdot 4a^2 \cdot 3b + 3\cdot 2a \cdot 9b^2 - 27b^3$
$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$
$12)(3x-5y)^3 = (3x)^3 - 3\cdot(3x)^2 \cdot 5y + 3\cdot 3x \cdot (5y)^2 - (5y)^3 =$
$= 27x^3 - 3\cdot 9x^2 \cdot 5y + 3\cdot 3x \cdot 25y^2 - 125y^3 = 27x^3 - 135x^2y + 225xy^2 - 125y^3$

$13)(5x+2y)^3 = (5x)^3 + 3\cdot(5x)^2 \cdot 2y + 3\cdot 5x \cdot (2y)^2 + (2y)^3 =$
$125x^3 + 3\cdot 25x^2 \cdot 2y + 3\cdot 5x \cdot 4y^2 + 8y^3 = 125x^3 + 150x^2y + 60xy^2 + 8y^3$

$14)(ab-5c)^3=(ab)^3-3\cdot(ab)^2\cdot 5c+3\cdot ab\cdot(5c)^2-(5c)^3=a^3b^3-3\cdot a^2b^2\cdot 5c+3\cdot ab\cdot 25c^2-125c^3$
$=a^3b^3-15a^2b^2c+75abc^2-125c^3$

$15)(3ab+5c)^3=(3ab)^3+3\cdot(3ab)^2\cdot 5c+3\cdot 3ab\cdot(5c)^2+(5c)^3=$
$=27a^3b^3+3\cdot 9a^2b^2\cdot 5c+3\cdot 3ab\cdot 25c^2+125c^3=27a^3b^3+135a^2b^2c+225abc^2+125c^3$

Napomena:Pravilo $(a\cdot b)^n=a^n\cdot b^n$ se može poopštiti za proizvoljan broj faktora tj.vrijedi
$(a\cdot b\cdot c)^n=a^n\cdot b^n\cdot c^n,....,(a\cdot b\cdot c\cdot ...\cdot m)^n=a^n\cdot b^n\cdot c^n\cdot ...\cdot m^n$
U našem slucaju (zad.15)imali smo: $(3ab)^3=3^3a^3b^3=27a^3b^3$

$16)(x^2+1)^3=(x^2)^3+3\cdot(x^2)^2\cdot 1+3\cdot x^2\cdot 1^2+1^3=x^6+3x^4+3x^2+1$

$17)\left(x-\frac{1}{3}\right)^3=x^3-3\cdot x^2\cdot\frac{1}{3}+3\cdot x\cdot\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^3=x^3-x^2+3\cdot x\cdot\frac{1}{9}-\frac{1}{27}=$
$=x^3-x^2+\frac{x}{3}-\frac{1}{27}$

$18)(x^3-y^3)^3=(x^3)^3-3\cdot(x^3)^2\cdot y^3+3\cdot x^3\cdot(y^3)^2-(y^3)^3=x^9-3x^6y^3+3x^3y^6-y^9$

$19)\left(2x+\frac{1}{2}\right)^3=(2x)^3+3\cdot(2x)^2\cdot\frac{1}{2}+3\cdot 2x\cdot\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3=8x^3+3\cdot 4x^2\cdot\frac{1}{2}+3\cdot 2x\cdot\frac{1}{4}+\frac{1}{8}=$
$=8x^3+6x^2+\frac{3}{2}x+\frac{1}{8}$

$20)\left(\frac{1}{3}ab+\frac{1}{4}c\right)^3=\left(\frac{1}{3}ab\right)^3+3\cdot\left(\frac{1}{3}ab\right)^2\cdot\frac{1}{4}c+3\cdot\frac{1}{3}ab\cdot\left(\frac{1}{4}c\right)^2+\left(\frac{1}{4}c\right)^3=$
$=\frac{1}{27}a^3b^3+3\cdot\frac{1}{9}a^2b^2\cdot\frac{1}{4}c+3\cdot\frac{1}{3}ab\cdot\frac{1}{16}c^2+\frac{1}{64}c^3=$
$=\frac{1}{27}a^3b^3+\frac{1}{12}a^2b^2c+\frac{1}{16}abc^2+\frac{1}{64}c^3$

$21)\left(a^2b^3+\frac{1}{3}\right)^3=\left(a^2b^3\right)^3+3\cdot\left(a^2b^3\right)^2\cdot\frac{1}{3}+3\cdot a^2b^3\cdot\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3=$
$=\left(a^2\right)^3\cdot\left(b^3\right)^3+3\cdot\left(a^2\right)^2\cdot\left(b^3\right)^2\cdot\frac{1}{3}+3\cdot a^2b^3\cdot\frac{1}{9}+\frac{1}{27}=$
$=a^6b^9+3\cdot a^4b^6\cdot\frac{1}{3}+3\cdot a^2b^3\cdot\frac{1}{9}+\frac{1}{27}=a^6b^9+a^4b^6+\frac{1}{3}a^2b^3+\frac{1}{27}$

$$22)\left(\frac{3}{4}x^2-\frac{4}{3}y^3\right)^3=\left(\frac{3}{4}x^2\right)^3-3\cdot\left(\frac{3}{4}x^2\right)^2\cdot\frac{4}{3}y^3+3\cdot\frac{3}{4}x^2\cdot\left(\frac{4}{3}y^3\right)^2-\left(\frac{4}{3}y^3\right)^3=$$
$$=\left(\frac{3}{4}\right)^3\cdot\left(x^2\right)^3-3\cdot\left(\frac{3}{4}\right)^2\cdot\left(x^2\right)^2\cdot\frac{4}{3}y^3+3\cdot\frac{3}{4}x^2\cdot\left(\frac{4}{3}\right)^2\cdot\left(y^3\right)^2-\left(\frac{4}{3}\right)^3\cdot\left(y^3\right)^3=$$
$$=\frac{27}{64}x^6-3\cdot\frac{9}{16}x^4\cdot\frac{4}{3}y^3+3\cdot\frac{3}{4}x^2\cdot\frac{16}{9}\cdot y^6-\frac{64}{27}y^9=\frac{27}{64}x^6-\frac{9}{4}x^4y^3+4x^2y^6-\frac{64}{27}y^9$$

$$23)\left(xy^2+z^3\right)^3=\left(xy^2\right)^3+3\cdot\left(xy^2\right)^2\cdot z^3+3\cdot xy^2\cdot\left(z^3\right)^2+\left(z^3\right)^3=$$
$$=x^3\cdot\left(y^2\right)^3+3\cdot x^2\cdot\left(y^2\right)^2\cdot z^3+3\cdot xy^2\cdot\left(z^3\right)^2+\left(z^3\right)^3=x^3y^6+3x^2y^4z^3+3xy^2z^6+z^9$$

$$24)\left(xy^3-z^4\right)^3=\left(xy^3\right)^3-3\cdot\left(xy^3\right)^2\cdot z^4+3\cdot xy^3\cdot\left(z^4\right)^2-\left(z^4\right)^3=$$
$$=x^3\cdot\left(y^3\right)^3-3\cdot x^2\cdot\left(y^3\right)^2\cdot z^4+3\cdot xy^3\cdot z^8-z^{12}=x^3y^9-3x^2y^6z^4+3xy^3z^8-z^{12}$$

$$25)\left(x^4y^2-z^3\right)^3=\left(x^4y^2\right)^3-3\cdot\left(x^4y^2\right)^2\cdot z^3+3\cdot x^4y^2\cdot\left(z^3\right)^2-\left(z^3\right)^3=$$
$$=\left(x^4\right)^3\cdot\left(y^2\right)^3-3\cdot\left(x^4\right)^2\cdot\left(y^2\right)^2\cdot z^3+3\cdot x^4y^2\cdot z^6-z^9=x^{12}y^6-3x^8y^4z^3+3x^4y^2z^6-z^9$$

$$26)\left(x^2y^3-3xy^2\right)^3=\left(x^2y^3\right)^3-3\cdot\left(x^2y^3\right)^2\cdot3xy^2+3\cdot x^2y^3\cdot\left(3xy^2\right)^2-\left(3xy^2\right)^3=$$
$$=\left(x^2\right)^3\cdot\left(y^3\right)^3-3\cdot\left(x^2\right)^2\cdot\left(y^3\right)^2\cdot3xy^2+3\cdot x^2y^3\cdot3^2\cdot x^2\cdot\left(y^2\right)^2-3^3\cdot x^3\cdot\left(y^2\right)^3=$$
$$=x^6y^9-3\cdot x^4\cdot y^6\cdot3xy^2+3\cdot x^2y^3\cdot9\cdot x^2\cdot y^4-27\cdot x^3\cdot y^6=x^6y^9-9x^5y^8+27x^4y^7-27x^3y^6$$
$$27)\left(5x^2+y^3z^4\right)^3=\left(5x^2\right)^3+3\cdot\left(5x^2\right)^2\cdot y^3z^4+3\cdot5x^2\cdot\left(y^3z^4\right)^2+\left(y^3z^4\right)^3=$$
$$=5^3\cdot\left(x^2\right)^3+3\cdot5^2\cdot\left(x^2\right)^2\cdot y^3z^4+3\cdot5x^2\cdot\left(y^3\right)^2\cdot\left(z^4\right)^2+\left(y^3\right)^3\cdot\left(z^4\right)^3=$$
$$125x^6+75x^4y^3z^4+15x^2y^6z^8+y^9z^{12}$$

$$28)\left(x^{-1}+x\right)^3=\left(\frac{1}{x}+x\right)^3=\left(\frac{1}{x}\right)^3+3\cdot\left(\frac{1}{x}\right)^2\cdot x+3\cdot\frac{1}{x}\cdot x^2+x^3=\frac{1}{x^3}+3\cdot\frac{1}{x^2}\cdot x+3x+x^3=$$
$$=\frac{1}{x^3}+\frac{3}{x}+3x+x^3$$

$$29)\left(x^{-3}+x^2\right)^3=\left(\frac{1}{x^3}+x^2\right)^3=\left(\frac{1}{x^3}\right)^3+3\cdot\left(\frac{1}{x^3}\right)^2\cdot x^2+3\cdot\left(\frac{1}{x^3}\right)\cdot\left(x^2\right)^2+\left(x^2\right)^3=$$
$$=\frac{1}{x^9}+3\cdot\frac{1}{x^6}\cdot x^2+3\cdot\frac{1}{x^3}\cdot x^4+x^6=\frac{1}{x^9}+\frac{3}{x^4}+3x+x^6$$

$30)\left(2^m-2^n\right)^3=\left(2^m\right)^3-3\cdot\left(2^m\right)^2\cdot 2^n+3\cdot 2^m\cdot\left(2^n\right)^2-\left(2^n\right)^3=2^{3m}-3\cdot 2^{2m}\cdot 2^n+3\cdot 2^m\cdot 2^{2n}-2^{3n}$
$=2^{3m}-3\cdot 2^{2m+n}+3\cdot 2^{m+2n}-2^{3n}$
$31)\left(a^n+b^n\right)^3=\left(a^n\right)^3+3\cdot\left(a^n\right)^2\cdot b^n+3\cdot a^n\cdot\left(b^n\right)^2+\left(b^n\right)^3=a^{3n}+3a^{2n}b^n+3a^nb^{2n}+b^{3n}$

$32)(a-1)^3+(a+1)^3=a^3-3a^2+3a-1+a^3+3a^2+3a+1=$
$=a^3+a^3-3a^2+3a^2+3a+3a-1+1=2a^3+6a$

$33)2(x+3)^3-3(x-1)^3=2\left(x^3+9x^2+27x+27\right)-3\left(x^3-3x^2+3x-1\right)=$
$=2x^3+18x^2+54x+54-3x^3+9x^2-9x+3=-x^3+27x^2+45x+57$

$34)(x-1)^3-x(x+2)^2=x^3-3x^2+3x-1-x\left(x^2+4x+4\right)=x^3-3x^2+3x-1-x^3-4x^2-4x=$
$=-7x^2-x-1$

$35)(x+y+z)^3=[(x+y)+z]^3=(x+y)^3+3\cdot(x+y)^2\cdot z+3\cdot(x+y)\cdot z^2+z^3=$
$=x^3+3x^2y+3xy^2+y^3+3\left(x^2+2xy+y^2\right)z+3(x+y)z^2+z^3=$
$=x^3+3x^2y+3xy^2+y^3+3x^2z+6xyz+3y^2z+3xz^2+3yz^2+z^3=$
$=x^3+y^3+z^3+6xyz+3x^2y+3xy^2+3x^2z+3xz^2+3y^2z+3yz^2$

Zadaci za samostalan rad učenika

$1)(a-2)^3=$

$2)(a-x)^3=$

$3)(2a-1)^3=$

$3)(u-v)^3=$

$4)(3x-y)^3=$

5) $(3x-7y)^3 =$

6) $\left(x^2-y^3\right)^3 =$

7) $\left(3x-\frac{1}{3}\right)^3 =$

8) $\left(\frac{1}{2}x+\frac{2}{3}\right)^3 =$

9) $\left(0{,}2x+\frac{5}{6}y\right)^3 =$

10) $(2x+0{,}1)^3 =$

11) $\left(2x^2y^3+3\right)^3 =$

12) $\left(ab^4+a^2b^3\right)^3 =$

13) $\left(a^2+2a^3b^2\right)^3 =$

14) $\left(3a^2-2b^3c^5\right)^3 =$

15) $\left(5x^2+y^3z^4\right)^3 =$

16 $\left(x^{-2}-x\right)^3 =$

17) $\left(3^n-2^n\right)^3 =$

18) $\left(2^n+3^n\right)^3 =$

19) $\left(a^n-a^{n+1}\right)^3 =$

20) $\left(a^{n+1}+a^{n-1}\right)^3 =$

21) $(x+y)^3-(x-y)^3 =$

22) $(x+y)^3+(x-y)^3 =$

23) $3(x+1)^3-5(2-x)^3 =$

24) $3(2x+y)^3-4(x+2y)^3 =$

25) $x(x-2)^2-(x+2)^3 =$

26) $2(x+y)^3-5(x-y)^3-\left(x^3+y^3\right)=$

Rješenje nekih zadataka za samostalan rad

6) $x^6-3x^4y^3+3x^2y^6-y^9 =$

7) $27x^3-9x^2+x-\frac{1}{27}$

8) $\frac{1}{8}x^3+\frac{1}{2}x^2+\frac{2}{3}x+\frac{8}{27}$

11) $8x^6y^9+36x^4y^6+54x^2y^3+27$

12) $a^3b^{12}+3a^3b^{11}+3a^5b^{10}+a^6b^9$

16) $\frac{1}{x^6}+\frac{3}{x^3}+3+x^3$